A VECTOR MEDIAN FILTER FOR HYPERSPECTRAL IMAGES BASED ON LEXICOGRAPHIC ORDERING OF ESTIMATED AUTO-CORRELATION FUNCTIONS

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ABSTRACT

We propose a vector median filter for hyperspectral images based on a ranking of pixel spectral values. The ranking of the pixel values is performed in the Fourier domain, based on the hypothesis that these values represent power spectral density functions of unknown random processes, under the assumption that these underlying random processes are stationary. The employed ordering scheme is a classical lexicographic one. We analyze the displacement maps and compare the proposed vector median filter against a classical vector median filter extended to the hyperspectral image case. We present and discuss the experimental results and then draw conclusions.

***Index Terms*—** vector median filter, spectra ranking, power spectral density function, Wiener-Khinchin theorem, autocorrelation function

1. INTRODUCTION

Vector median filters (VMFs) are deployed for de-noising of images affected by impulsive noise, as the image acquisition process is inevitably affected by noise. Thus, their main purpose is to recover as much as possible from the original signal. In the case of median filtering for multi-dimensional images, the approaches must take into account the multivariate or vector nature of the data, as well as the spectral variability in the particular case of hyperspectral images. VMFs are very popular and relatively easy to implement non-linear approaches. They require the definition of an ordering scheme for the original vector pixel values in a specified filtering window. There exist various approaches for vector data ordering schemes, which have been classified in four groups according to [1]: marginal, reduced, partial and conditional, each of them exhibiting both advantages and disadvantages. For instance, the marginal ordering scheme for color images does not consider the spectral correlation between color channels and introduces false colors; the reduced and partial orderings rely on pre-orderings, thus they lack the property of antisymmetry, or generate perceptual non-linearities due to their behavior similar to conditional orderings [2].

A series of VMFs have been designed, mostly for applications on color images [3]. The approaches include: basic vector filters (Vector Median Filter, Extended Vector Median Filter [4] [5] or Directional Vector Median Filter [6]), weighted vector filters (Weighted Vector Median Filter [7] or Rank Order Weighted Vector Median Filter [8]) and adaptive vector filters (Adaptive Vector Median filter [9] or Adaptive based Impulsive Noise Removal Filter [10]). Various non-linear filtering approaches based on mathematical morphology exist for color and multivariate images [11], as the mathematical morphology framework is also based on imposing an ordering relation on the data to be filtered. Several morphological frameworks for vectorial data have been proposed more recently, such as a method based on a fuzzy generalization of mathematical morphology for color data [12] and two approaches based on a reduced ordering [13]. The lexicographic ordering scheme, in particular, has been used in pseudo-morphological approaches such as the α-trimmed Pseudo-Morphology [14] and the Maximum Distance Pseudo-Morphology [15].

Various factors have a negative impact on the quality of remotely-sensed hyperspectral images. The most important is the spectral mixing due to low spatial resolution, resulting in spectral variability which is addressed by a plethora of spectral unmixing approaches [16], among the most recent ones being the improved linear mixing models like [17] or deep learning-based [18]. Another factor affecting the geometric accuracy of high-resolution satellite images is attitude jitter, caused by the spatial environment and spacecraft instability during data acquisition [19], with a series of correction methods having been proposed recently [20] [21] [22].

In this paper, we propose a vector median filter for hyperspectral images based on the ranking of hyper-spectral image pixel values modelled as power spectral density functions [23] and considering a lexicographic ordering resulting in a global ordering [24]. In order to compare the behavior of the proposed filter, we extend the classical VMF proposed by Astola [25] to the case of hyperspectral images. The contributions are: an ordering performed in the Fourier domain and a lower computational burden compared to a classical VMF. In Section 3 we show experimental results, comparison and discuss the applicability of the proposed VMF on a widely-known hyperspectral image. In Section 4 we draw the conclusions.

2. APPROACH

The Wiener-Khinchin theorem [26] states that the power spectral density function $q_\xi(\omega)$ of a wide-sense-stationary random process ξ is the Fourier transform of its autocorrelation function $R_\xi(\tau)$. Consequently, considering that the hyperspectral image pixel values are power spectral density functions of an unknown random process $\xi(t)$ and, in addition, continuous functions of the wavelength λ, and making the assumption that the unknown random process $\xi(t)$ is a wide-sense stationary random process, thus the power spectral density function does exist and it is a Fourier pair with the autocorrelation function:

$$R_\xi(\tau) = \mathcal{F}^{-1}\{q_\xi(\lambda)\} = \int_{-\infty}^{+\infty} q_\xi(\lambda) e^{+j\lambda\tau} d\lambda \tag{1}$$

Thus, we compute the inverse Fourier transform of the hyperspectral image pixel values to retrieve an estimate of the autocorrelation function $R_\xi(\tau)$. In order to design our vector median filter for hyperspectral images, we define a lexicographic order for the values of the estimated autocorrelation functions $R_1(\tau)$ and $R_2(\tau)$ for any two pixels P_1 and P_2 in the hyperspectral image as follows:

$$P_1 < P_2 \Leftrightarrow \tag{2}$$

$$\begin{cases} R_1(0) < R_2(0), \\ R_1(0) = R_2(0) \& R_1(1) < R_2(1), \\ R_1(0) = R_2(0) \& R_1(1) = R_2(1) \& R_1(2) < R_2(2), \\ \dots \end{cases}$$

The ordering is enforced in the Fourier domain. In order to apply the proposed ordering for color pixels in the frame of a vector median filter, we consider the implementation of the filter in a common 3×3 window with the origin in the center.

For comparison, we chose the VMF proposed by Astola [25], which we briefly remind here. In [25] the median vector is proved to correspond to the one which minimizes the sum of distances from one pixels to the others from the filtering window. If within the analysis window there are N pixels denoted $P_1, P_2, P_3, \cdots, P_N$, the median M is the one minimizing the cumulative distance to all the other $N-1$ pixels:

$$M = \arg\min_i D_i = \sum_{j=1}^{N} d(P_i - P_j) \tag{3}$$

In order to extend this approach to the case of hyperspectral images, the distance d in eq. (3) must be specified appropriately. We deployed two distances: the spectral angle measure (SAM) [27] and the Euclidian distance.

3. EXPERIMENTAL RESULTS

In this section we analyze the behavior of the proposed VMF, comparing it against the Astola VMF extended to hyperspectral images, from the point of view of the displacements induced in the pixel position due to the deployed ordering scheme and the choice of the median vector value. We consider a widely-known hyperspectral image, Pavia University, consisting of 103 spectral bands, captured by the ROSIS sensor. In Fig. 1 we illustrate the color mapping of the considered displacements, assuming a parallax disparity of each pixel possibly in both along-track and cross-track directions: black for the pixels that remain in the same position after the median filtering, yellow for pixels that were displaced from neighboring pixels situated at a spatial distance of 1 and red for the pixels moved from locations at a $\sqrt{2}$ spatial distance. The displacement maps were produced using this color mapping scheme. Other color mappings can be defined, based on the scheme deemed necessary in choosing the VMF to be employed in the respective application.

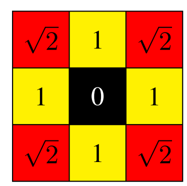


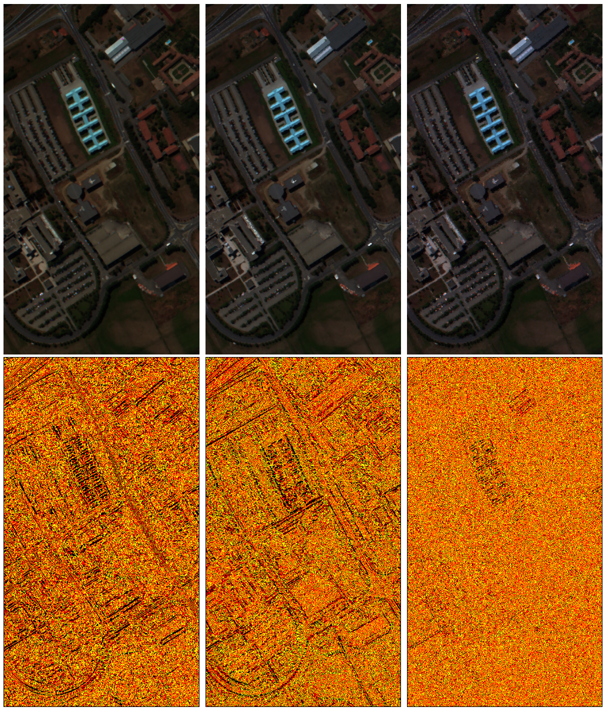
Fig. 1. Color mapping of the 3×3 vicinity.

In Fig. 2 we show the median fitering results on the Pavia University image, with the corresponding pixel displacement maps. One may notice that the results obtained with the proposed VMF are very similar with the ones obtained with Astola VMF using SAM and that for both of them most of the pixels that remained on the same position (black pixels) are usually located along the image edges. For the Astola VMF using Euclidian distance, the displacements are more or less random with uniform distribution, except for the regions corresponding to the painted metal sheets. For clarity, the results on two crops of the image are depicted in Figs. 3 and 4.

In Table 1 we show the percentages of black, yellow and red pixels in the displacement maps in Fig. 2, showing once more the similar behavior between the proposed and SAM-based Astola VMFs. Table 2 depicts the percentage of matched pixels between pairs of displacement maps in Fig. 2; basically we compute the percentage of pixels which have the same color (black, yellow or red) in the displacement maps.

	0	1	$\sqrt{2}$
Proposed VMF	25.19 %	37.71 %	37.10 %
Astola SAM	24.59 %	39.87 %	35.53 %
Astola Euclid.	13.22 %	44.19 %	42.59 %

Table 1. Percentage of displaced pixels for Pavia Univ.



(a) Proposed VMF (b) Astola SAM (c) Astola Euclid.

Fig. 2. Experimental results on Pavia Univ.: images (top) and displacement maps (bottom).

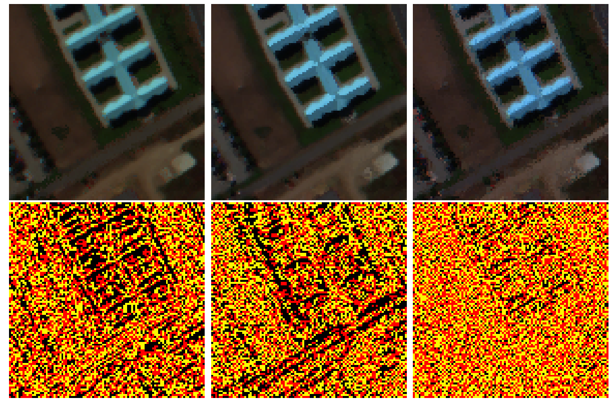
In order to assess the quality of the filtered images, we employed the Structural Similarity Index Metric (SSIM) [28], computed on the color versions, using the unfiltered image as reference. The values obtained on the Pavia University image are presented in Table 3. They indicate that the proposed VMF produces the result that is the closest perceptually to the original, when visualized as a color image.

4. CONCLUSIONS AND FUTURE WORK

We proposed a vector median filter for hyperspectral images based on the pixel spectral values ranking, considering that the hyperspectral image pixel values represent power spectral density functions of unknown random processes. Assuming their stationarity, we invoked the Wiener-Khinchin theorem for retrieving the corresponding autocorrelation functions through the inverse Fourier transform. We then imposed a lexicographic order to the autocorrelation functions. We compared the proposed vector median filter against a classical vector median filter extended to the hyperspectral image case. We investigated the behavior of the proposed filter in terms of statistics of displacements and structural similarity. We report on the experimental results and conclude that the proposed VMF can be a valid candidate and the displacement maps can be an effective tool for choosing the VMF for the considered application. As future work, we shall investigate the capabilities of the proposed filter in a denoising or attitude jitter correction application, as well as a pre-processing stage for spectral variability reduction for unmixing approaches.

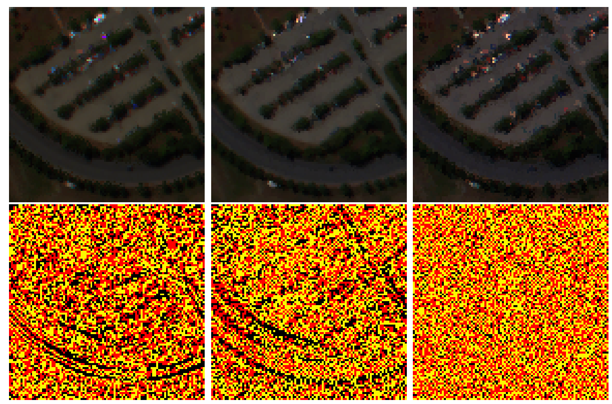
Matching	0	1	$\sqrt{2}$	Total
P.VMF, A.SAM	7.11%	13.86%	15.57%	36.54%
P.VMF, A.Euclid.	3.75%	16.97%	16.06%	36.78%
A.SAM, A.Euclid.	5.80%	21.25%	18.49%	45.54%

Table 2. Percentage of identical pixels between pairs of displacement maps for Pavia Univ.



(a) Proposed VMF (b) Astola SAM (c) Astola Euclid.

Fig. 3. Experimental results on Pavia Univ. (crop1): images (top) and displacement maps (bottom).



(a) Proposed VMF (b) Astola SAM (c) Astola Euclid.

Fig. 4. Experimental results on Pavia Univ. (crop2): images (top) and displacement maps (bottom).

	SSIM
Proposed VMF	0.9193
Astola SAM	0.9043
Astola Euclid.	0.8723

Table 3. SSIM values on Pavia Univ.

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